Visualising Special Relativity

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Abstract
We describe a graphics package we have developed for producing photo-realistic images of relativistically moving objects. The physics of relativistic images is outlined.

INTRODUCTION

Since Einstein's first paper on relativity[3] physicists have wondered how things would *look* at relativistic speeds. However it is only in the last forty years that the physics of relativistic images has become clear. The pioneering studies of Penrose[4], Terrell[5], and Weisskopf[6] showed that there is more to relativistic images than length contraction. At relativistic speeds a rich visual environment is produced by the combined effects of the finite speed of light, aberration, the Doppler effect, time dilation, and length contraction.

Computers allow us to use relativistic physics to construct realistic images of relativistic scenes. There have been two approaches to generating relativistic images: ray tracing[7,8] and polygon rendering[9,10,11]. Ray tracing is slow but accurate, while polygon rendering is fast enough to work in real time[11], but unsuitable for incorporating the full complexity of everyday scenes, such as shadows. One of us (A.S.) has developed a ray tracer, called "Backlight"[12], which maximises realism and flexibility.

Relativity is difficult, at least in part, because it challenges our fundamental notions of space and time in a disturbing way. Nevertheless, relativistic images, such as Figure 1, are dramatic but comprehensible. It is part of most people's experience that curved mirrors and the like can produce strange optical distortions.

Figure 1, and other images in this paper, are from a video we have produced. The purpose of the video is to illustrate relativistic optical effects on familiar objects. The only fiction we must introduce to produce scenes like Figure 1 is the slowing down of the speed of light, to about c=5m/s for most of our scenes. Changing the speed of light acts like a scaling. It does not change any of the physics, just the space-time scale for which the effects are significant. Alternatively, we could regard the objects in our scenes as enormously large. We have used these videos for several years in the relativity section of the ANU first year physics course. Copies may be ordered from the project web site[12]. The video screening accompanies a lecture introducing aberration and simple relativistic optics. We are currently adapting developing animations for the ANU Wedge walk-in virtual reality theatre[13].

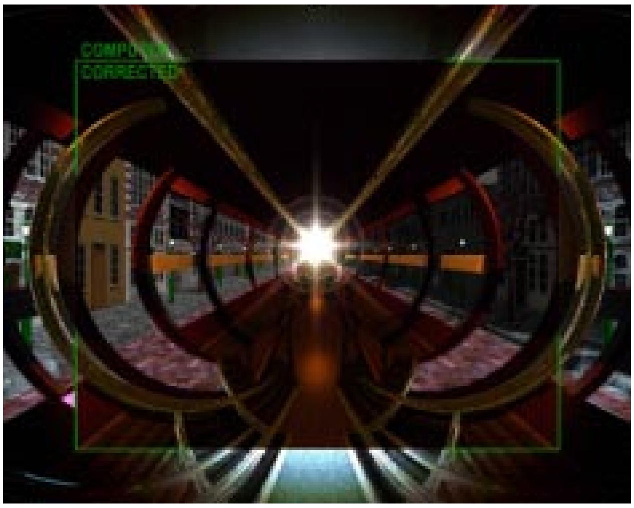


Figure 1. Image from a camera flying through the inside of a tram at the relativistic speed 0.9c. The central portion has had the Doppler and headlight effects removed to highlight the geometrical effects. An animation of this image is on the paper's website[14].

RELATIVISTIC OPTICS

Images of relativistically moving objects may be constructed by Lorentz transforming an object’s world line into the camera frame and allowing for the finite propagation speed of light. The Lorentz transformation generates a length contraction, which makes the object shorter than its rest length, in the direction of its motion. The finite speed of light , or light delay, generates important additional effects.

For example, a line perpendicular to the direction of motion is not length contracted. However the camera receives light from the closest part of the line first. Light from the more distant parts of the line takes longer to reach the camera and hence must be emitted earlier. Consequently, the line's image is curved[16], as can be seen in the tram window frames in Figure 1.

Unfortunately some authors, including Gamow in *Mr. Tompkins in Wonderland*[15], have ignored the effects of the finite speed of light. Because of the light delay, a pure length contraction is seen only under special circumstances. For example, consider an object moving along a line offset from the camera, such as the tram in Figure 2. Let the object have negligible thickness in the direction from the object to the camera. When a light ray from the object to the camera makes a right angle with the object's direction of motion, in the camera frame, the length contraction dominates in a small solid angle around the ray[16].

By contrast, such an object moving away from the camera is contracted by light delay effects, as well as by length contraction. This is because the closer parts emit later, when the object as a whole has moved further away. Similarly, offset objects moving towards the camera are expanded by light delay effects. These effects are illustrated in Figure 2.

(a) (b)

Figure 2. Length contraction and Terrell rotation. (a) Tram at rest in the camera, and street, frame. (b) Tram moving from right to left at 0.866c. The Terrell rotation makes the rear of the tram visible. Animation on web site[14].

Furthermore, it may be possible to see the trailing side of objects, which is impossible at low velocities. This is because an object whose velocity exceeds that of the projection of the light ray's velocity in the direction of the object's motion “gets out of the way” of the light and hence does not block it. An animation illustrating this effect is available on the paper's website[14]. For similar reasons the leading side of an offset oncoming object, which could be seen at low velocity, may be invisible at sufficiently high velocities. For objects subtending a small solid angle to the camera, the combined result of the Lorentz transformations and light delay mimic a rotation of the object. This is known as Terrell rotation[5], and is illustrated in Figure 2. When the camera images a small object in direction making an angle β relative to the direction of its motion (with a directly incoming object viewed at angle $\beta=0$) the object appears approximately as it would in its rest frame when viewed from the aberrated angle α, given by Eq.(1) below. That is, it appears rotated through the angle α-β (towards the velocity vector).

Interestingly, Penrose[4] noted that a sphere always presents a circular outline, not just under the specific conditions for Terrell rotation. This can be seen in Figure 3 which shows the view from Earth orbit at relativistic speed.

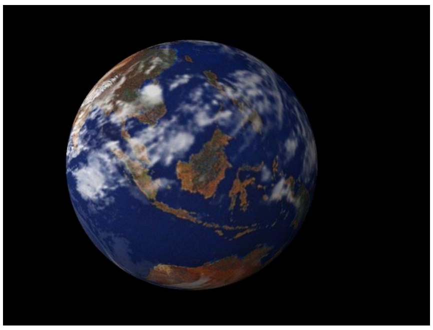


Figure 3. The Earth as seen at a speed of 0.95c from an orbit height of about 0.5 Earth radii. Note the circular outline of the Earth. Australia is visible at the bottom. Animation on web site[14].

Terrell rotation may be understood in terms of length contraction and light delay, or in terms of relativistic aberration. Relativistic aberration is analogous to Newtonian aberration, a familiar example of which is the decrease in the incident angle of rain when driving through it. Consider a light ray coming in at angle α to the velocity vector of another frame moving with speed v , in which the light ray comes in with angle β. Defining the zero angle to be that of an oncoming ray, these angles are related by the relativistic aberration formula[17]

$$\tan\left(\tfrac{1}{2}\beta\right) = \left(\frac{c-v}{c+v}\right)^{1/2} \tan\left(\tfrac{1}{2}\alpha\right) \tag{1}$$

This relation is graphed in Figure 4. The factor relating the tangents is always less than one, so the angles of incoming light rays are rotated towards the direction of motion. Angles for which the slope of the graph is less than one are compressed, while those for which the slope is greater than one are expanded. For example, for *v/c*=0.99 angles less than about 2.6 radians are compressed. This means that objects within this view angle in their rest frame will appear smaller, while objects at greater view angles will appear magnified. These effects can be seen in Figure 5.

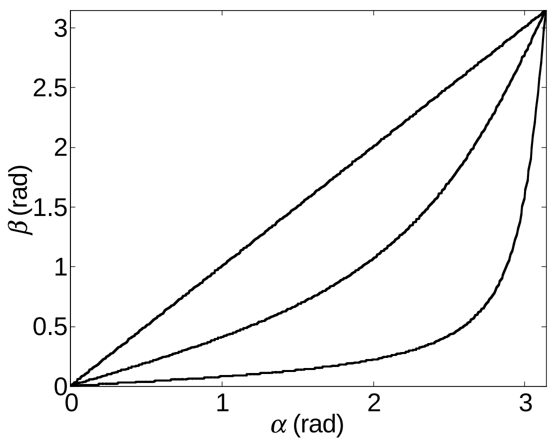
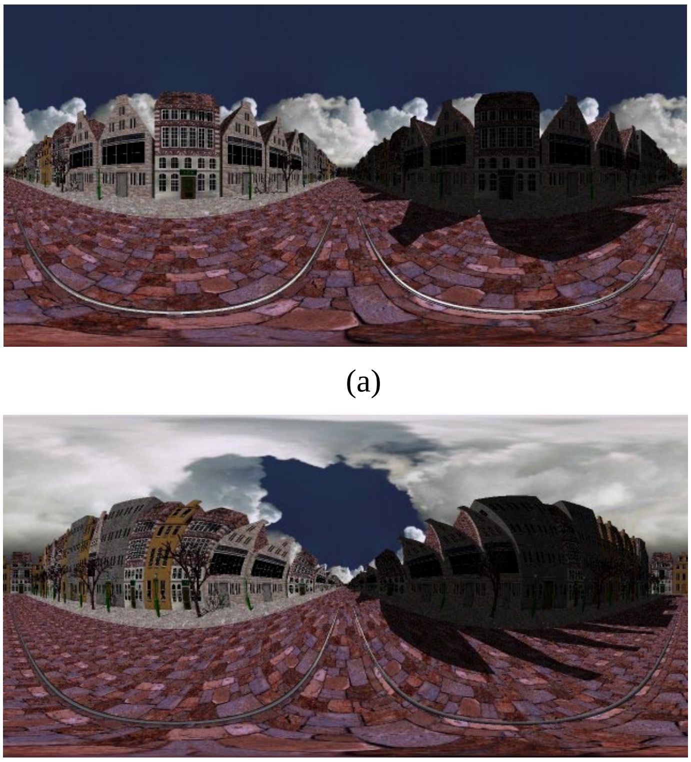


Figure 4. Graph of the aberrated angle according to Eq.(1). *v/c* = 0 (top), 0.75 (middle), 0.99 (bottom).

This aberration formula determines the appearance of any relativistic scene. It transforms the light rays making up the image between frames, irrespective of their origin. Figure 6 illustrates the aberration effect on a star field seen by observer moving through its rest frame. The distant stars bunch up in front of the observer, while the view behind the ship becomes empty of stars. As the speed of light is approached a bright spot of compressed stars is seen directly in front of the observer. The inverse of this is known as the "headlight effect". This refers to the radiation from a moving object being directed into a narrow forward cone, for example in synchrotron light sources.



(a)

(b)

Figure 5. Mercator projections of a street scene from an observer moving down the street. The left and right edges are the image behind the observer. (a) The camera at rest in the street. (b) The camera moving at the relativistic speed 0.9c. Almost all of the cloud corresponds to the portion above the street, behind the observer in (a). The rear view is therefore magnified while the forward view is compressed. Animation on website[14].

To fully understand relativistic images, three other physical effects must be considered; the Doppler effect, time dilation, and the aberration of solid angle. The Doppler effect shifts the frequency ω of light propagating in the direction with unit vector **d** to frequency ω' in the frame having relative velocity **v**, according to

$$\omega' = \omega\gamma(1 - \mathbf{v}\cdot\mathbf{d}/c) = \omega D, \qquad (2)$$

where we have defined the Doppler factor *D*. This shift is a first order effect in *v*/*c* and may become large well before other effects. For example at half the speed of light red shifted frequencies may be approximately halved. Hence the visible colour and intensity of a relativistic object may depend on its spectral properties, and those of the illumination sources, beyond the visible spectrum.

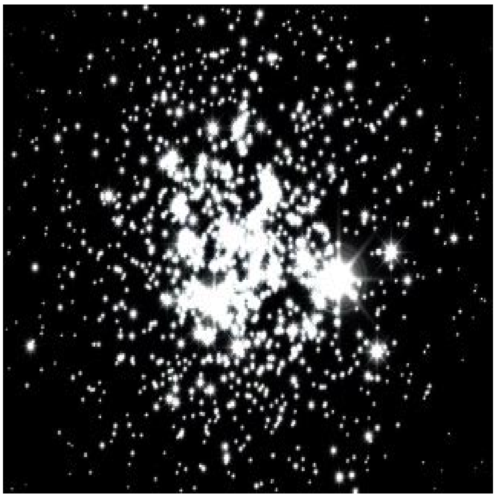


Figure 6. Image of a star field with camera moving at 0.9c through the stars' rest frame, in which they are isotropically distributed. Animation on website[14].

The Doppler shift changes the light intensity by the factor D, due to the proportionality between photon energy and frequency. Time dilation accounts for the different time intervals occupied by a group of N photons in different frames. It may be thought of as the dilation of the time for which the camera shutter is open, and changes the intensity by a factor of γ. Abberation compresses or expands the proper solid angle $d\Omega$ viewed by a camera pixel to $d\Omega'$. Since $d\Omega'/d\Omega = D^2$, this changes the intensity by an additional factor of D^2, independent of the object's spectral properties[18].

RELATIVISTIC IMAGES

Our ray tracer "Backlight" is more capable than previously reported relativistic ray tracers, such as that of Hsiung et al.[7] and Howard et al.[8], which, as far as we are aware, are not currently supported.

"Backlight" is written in ANSI C++ and has been compiled and run on a variety of platforms, including IBM-compatible PCs and UNIX systems. On a Silicon Graphics R10000 processor a minute of broadcast quality video for a simple scene took several days to render. Figure 7, with 720x526 pixel resolution took about 13 minutes to render on a 400 MHz Pentium II.

Polygon rendering has been used by Rau et al.[11] in a package which enables the user to fly through scenes at relativistic speeds, while changing speed and direction in real time. This is possible due to the hardware acceleration of polygon rendering now available on 3D graphics cards. Gekelman et al.[9] reported a very limited polygon renderer also capable of real time rendering, while Meng-Chou Chang et al.[10] described a powerful polygon renderer, which however lacks real time capability.

Polygon rendering is fast, even without hardware acceleration, because it projects onto the image only the vertices of the polygons constituting the objects. For ray tracing this must be done for every pixel. The image of the polygon surface is generated by straight line interpolation between the projected vertices. In non-relativistic rendering straight lines in the scene are imaged to straight lines. However in relativistic scenes straight lines image to curved lines, and hence the rendering algorithm may fail for high velocities. A solution is to finely mesh the objects, however unless the region of maximum distortion is known beforehand the entire scene must be finely meshed, at a large computational cost. In summary, ray tracing trades off the speed of polygon rendering for inherent physical accuracy. We now proceed to describe our relativistic ray tracer.

A relativistic image is made by those light rays arriving simultaneously at the camera from the surrounding scene. As usual in relativity there are different ways to proceed corresponding to the different inertial frames in which one may work. The relevant frames are the camera rest frame and the rest frames of objects and light sources in the scene.

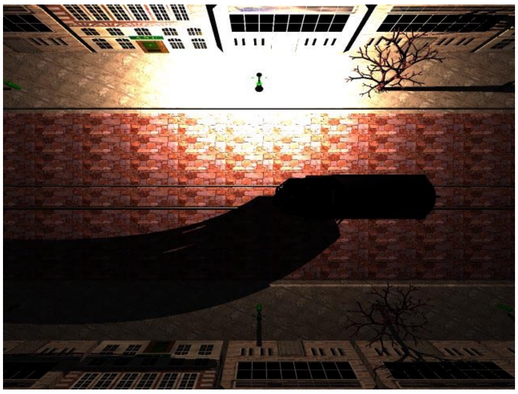


Figure 7. Shadow of a tram moving at 0.9c. Animation on website[14].

Our ray tracer models a pinhole camera. To generate an image we time-reverse the physical situation and trace hypothetical light rays backwards in time from each camera pixel to their events of origin. We then determine which of these hypothetical light rays correspond to actual light rays originating from objects in the scene. Light rays are Lorentz transformed between the camera fame and the rest frames of the various objects and lights sources in the scene.

In its rest frame the surface of an object is defined as the set of points satisfying an algebraic equation: $\{\mathbf{x}: f(\mathbf{x}) = 0\}$. For example, the surface of a unit sphere centred on the coordinate origin is defined by: $f(\mathbf{x}) = |\mathbf{x}|^2 - 1$. A light ray is specified by its propagation direction **d**, the originating camera location **p**, and the time of intersection with the pixel *T*. A ray intersects an object if the equation $f(\mathbf{p} + \mathbf{d}c(t - T)) = 0$ has a solution for some time $t<T$. The last inequality is necessary because we are propagating backwards in time from the camera to the object. If more than one intersection is found then the closest in time to the image time T is the required intersection. It does not matter which frame “closest” is calculated in, since time ordering along a light ray is the same in all frames.

Having found the object illuminating a pixel, we must determine the colour and intensity of the illumination. For this purpose we slightly extend the usual physical conception of a light ray to include intensity and colour information. For the case of direct illumination by a point light source a ray from the source to the object intersection is constructed.

Intersections with scene objects are tested for to account for shadows. The colour and intensity of the source are transformed into the object rest frame and combined with the reflectance properties of surface, using standard computer graphics techniques[19]. Finally the intensity and colour are transformed into the camera frame.

Since shadows behave in interesting ways[20] their straightforward handling by ray tracers is an important advantage. Polygon rendering requires the analytic calculation of the shapes of edge shadows. Ray tracing determines shadows in the same way as all illumination is determined: by tracing back from scene points to light sources. If some object is between a light source and the scene point, that point is in shadow. When an object, the light sources, and the camera are all in relative motion the shape and position of the object's shadow is influenced by the location and motion of the scene components. Figure 7 shows a characteristically curved shadow from a relativistically moving object.

CONCLUSIONS

The Backlight web site[12] includes animations of many of the figures in this paper, compressed versions of our instructional video, an order form for a VHS version of the video, and the downloadable Backlight code.

The full potential of computer graphics for teaching relativity has probably not been realised. It may be possible for students to use relativistic visualisation to learn relativity in a less abstract and more experiential way. The corresponding shift in pedagogic emphasis would be from the abstract "observer" to the concrete, and personal, "viewer".

As the cost effectiveness of computing decreases it should be possible to develop interactive laboratories, including virtual reality laboratories such as the ANU Wedge[13]. But even without that, the observation of pre-rendered relativistic images and animations might be used to help students "discover" relativistic physics. In this approach the learning of relativity might be based on understanding the physics of the images.

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