Chapter 6
LIGHT AND GEOMETRICAL OPTICS

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Reflection of Light

Light (along with all other forms of electromagnetic radiation) is a fundamental entity, and physics is still struggling to understand it. On an observable level, light manifests two seemingly contradictory behaviors, crudely pictured via wave and particle models. Usually the amount of energy present is so large that light behaves as if it were an ideal continuous wave, a wave of interdependent electric and magnetic fields. The interaction of light with lenses, mirrors, prisms, slits, and so forth, can satisfactorily be understood via

the wave model. On the other hand, when light is emitted or absorbed by the atoms of a system, these processes occur as if the radiant energy is in the form of minute, localized, well-directed blasts; that is, as if light is a stream of particles. Fortunately, without worrying about the very nature of light, we can predict its behavior in a wide range of practical situations.

Law of Reflection

A ray is a mathematical line drawn perpendicular to the wavefronts of a lightwave. It shows the direction of propagation of electromagnetic energy. In *specular* (or *mirror*) reflection, the angle of incidence equals the angle of reflection, as shown in Figure 6-1. Furthermore, the incident ray, reflected ray, and normal to the surface all lie in the same plane, called the *plane-of-incidence*.

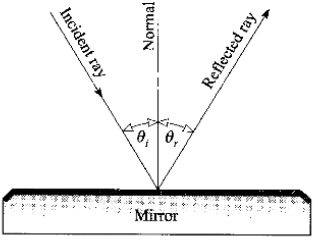

Figure 6-1

Plane Mirrors

Plane mirrors form images that are erect, of the same size as the object, and as far behind the reflecting surface as the object is in front of it. Such an image is *virtual*; i.e., the image will not appear on a screen located at the position on the image because the light does not converge there.

Spherical Mirrors

The *principal focus* of a spherical mirror, such as the ones shown in Figure 6-2, is the point F where rays parallel to and very close to the *central* or *optical axis* of the mirror are focused. This focus is real for a concave mirror and virtual for a convex mirror. It is located on the optical axis and midway between the center of curvature C and the mirror.

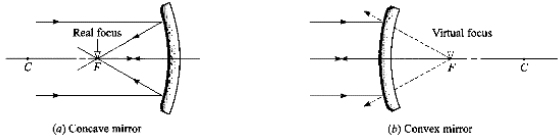

Figure 6-2

Concave mirrors form inverted real images of objects placed beyond the principal focus. If the object is between the principal focus and the mirror, the image is virtual, erect, and enlarged.

Convex mirrors produce only erect virtual images of objects placed in front of them. The images are diminished (smaller than the object) in size.

Mirror Equation

The **mirror equation** for both concave and convex spherical mirrors is

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{2}{R}=\frac{1}{f}$$

where s_o = object distance from the mirror
s_i = image distance from the mirror
R = radius of curvature of the mirror
f = focal length of the mirror = R/2

In addition,

- s_o is positive when the object is in front of the mirror.
- s_i is positive when the image is real, i.e., in front of the mirror.
- s_i is negative when the image is virtual, i.e., behind the mirror.
- R and f are positive for a concave mirror and negative for a convex mirror.

The **size of the image** formed by a spherical mirror is given by

$$\text{Linear magnification} = \frac{\text{length of image}}{\text{length of object}}$$

$$= \frac{\text{image distance from mirror}}{\text{object distance from mirror}} = \left|\frac{s_i}{s_o}\right|$$

Refraction of Light

Speed of Light

The **speed of light** as ordinarily measured varies from material to material. Light (treated macroscopically) travels fastest in vacuum, where its speed is $c = 2.998 \times 10^8$ m/s. Its speed in air is c/1.000 3. In water, its speed is c/1.33, and in ordinary glass it is about c/1.5. Nonetheless, on a microscopic level, light is composed of photons and photons exist only at the speed c. The apparent slowing down in material media arises from the absorption and re-emission as the light passes from atom to atom.

Index of Refraction

The **absolute index of refraction of a material** is defined as

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in the material}} = \frac{c}{v}$$

For any two materials, the **relative index of refraction** of material-1, with respect to material-2, is

$$\text{Relative index} = \frac{n_1}{n_2}$$

where n_1 and n_2 are the absolute refractive indices of the two materials.

Refraction

When a ray of light is transmitted obliquely through the boundary between two materials of unlike index of refraction, the ray bends. This phenomenon, called **refraction**, is shown in Figure 6-3.

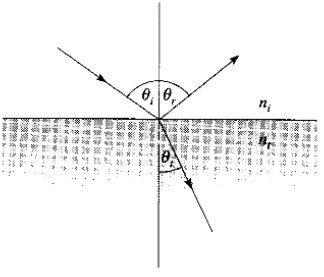

Figure 6-3

If $n_t > n_i$, the ray refracts as shown in the figure; it bends toward the normal as it enters the second material. If $n_t < n_i$, however, the ray refracts away from the normal. This would be the situation in Figure 6-3 if the direction of the ray were reversed. In either case, the incident and refracted (or transmitted) rays and the normal all lie in the same plane. The angles θ_i and θ_t in Figure 6-3 are called the *angle of incidence* and *angle of transmission* (or refraction), respectively.

Snell's Law

The way in which a ray refracts at an interface between materials with indices of refraction n_i and n_t is given by **Snell's Law**:

$$n_i \sin \theta_i = n_t \sin \theta_t$$

where θ_i and θ_t are as shown in Figure 6-3. Because this equation applies to light moving in either direction along the ray, a ray of light follows the same path when its direction is reversed.

Critical Angle for Total Internal Reflection

When light reflects off an interface where $n_i < n_t$, the process is called *external reflection*; when $n_i > n_t$, it is *internal reflection*. Suppose that a ray of light passes from a material of higher index of refraction to one of lower index, as shown in Figure 6-4.

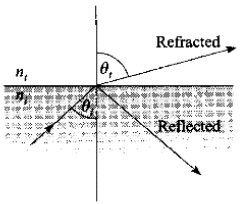

Figure 6-4

Part of the incident light is refracted and part is reflected at the interface. Because θ_t must be larger than θ_i, it is possible to make θ_i large enough so that $\theta_t = 90^o$. This value for θ_i is called the **critical angle** θ_c. For θ_i larger than this, no refracted ray can exist; all the light is reflected. The condition for total internal reflection is that θ_i exceed the critical angle θ_c, where

$$n_i \sin \theta_c = n_t \sin 90^o \quad \text{or} \quad \sin \theta_c = \frac{n_t}{n_i}$$

Because the sine of an angle can never be larger than unity, this relation confirms that total internal reflection can occur only if $n_i > n_t$.

Prism

A **prism** can be used to disperse light into its various colors, as shown in Figure 6-5. Because the index of refraction of a material varies with wavelength, different colors of light refract differently. In nearly all materials, red is refracted least and blue is refracted most.

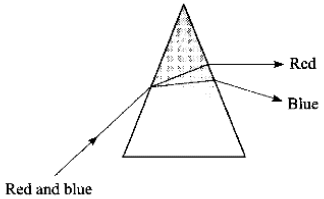

Figure 6-5

Thin Lenses

Types of Lenses

As indicated in Figure 6-6, *converging*, or *positive*, lenses are thicker at the center than at the rim and will converge a beam of parallel light to a real focus. *Diverging*, or *negative*, lenses are thinner at the center than at the rim and will diverge a beam of parallel light from a virtual focus.

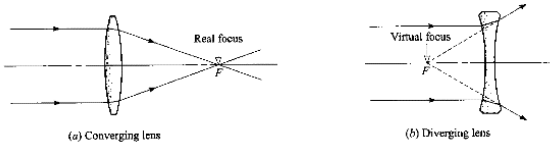

Figure 6-6

The *principal focus* (or *focal point*) of a thin lens with spherical surfaces is the point F where rays parallel to and near the central or optical axis are brought to a focus; this focus is real for a converging lens and virtual for a diverging lens. The *focal length* f is the distance of the principal focus from the lens. Because each lens in Figure 6-6 can be reversed without altering the rays, two symmetric focal points exist for each lens.

The object and image relation for converging and diverging lenses is

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$

where s_o is the object distance from the lens, s_i is the image distance from the lens, and f is the focal length of the lens. The lens is assumed to be thin, and the light rays *paraxial* (close to the principal axis). Then,

- s_o is positive for a real object, and negative for a virtual object.
- s_i is positive for a real image, and negative for a virtual image.
- f is positive for a converging lens, and negative for a diverging lens.

Also,

$$\text{Linear magnification} = \frac{\text{size of image}}{\text{size of object}}$$

$$= \frac{\text{image distance from lens}}{\text{object distance from lens}} = \left|\frac{s_i}{s_o}\right|$$

You Need to Know ✓

Converging lenses form inverted real images of objects located outside the principal focus. When the object is between the principal focus and the lens, the image is virtual (on the same side of the lens as the object), erect, and enlarged.

Diverging lenses produce only virtual, erect, and smaller images of real objects.

Lensmaker's Equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where n is the refractive index of the lens material, and r_1 and r_2 are the radii of curvature of the two lens surfaces. This equation holds for all types of thin lenses. A radius of curvature, r, is positive when its center of curvature lies to the right of the surface, and negative when its center of curvatures lies to the left of the surface.

If a lens with refractive index n_1 is immersed in a material with index n_2, then n in the lensmaker's equation is to be replaced by n_1/n_2.

Lens Power

Lens power in *diopters* (m^{-1}) is equal to 1/f, where f is the focal length expressed in meters.

Lenses in Contact

When two thin lenses having focal lengths f_1 and f_2 are in close contact, the focal length f of the combination is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

For lenses in close contact, the power of the combination is equal to the sum of their individual powers.

Optical Instruments

Combination of Thin Lenses

To locate the image produced by two lenses acting in combination,

(1) Compute the position of the image produced by the first lens alone, disregarding the second lens.

(2) Then consider this image as the object for the second lens, and locate its image as produced by the second lens alone.

This latter image is the required image.

If the image formed by the first lens alone is computed to be behind the second lens, then that image is a virtual object for the second lens, and its distance from the second lens is considered negative.

Important Point!

The Human Eye

The **human eye** uses a variable-focus lens to form an image on the retina at the rear of the eye. The *near point* of the eye, represented by d_n, is the closest distance to the eye from which an object can be viewed clearly. For the normal eye, d_n is about 25 cm. *Farsighted* persons can see distinctly only objects that are far from the eye; *nearsighted* persons can see distinctly only objects that are close to the eye.

Magnifying Glass

A **magnifying glass** is a converging lens used so that it forms an erect, enlarged, virtual image of an object placed inside its focal point. The magnification due to a magnifier with focal length f is $(d_n/f) + 1$ if the image it casts is at the near point. Alternatively, if the image is at infinity, the magnification is d_n/f.

Microscope

A **microscope** that consists of two converging lenses, an objective lens (focal length f_o) and an eyepiece lens (f_e), has

$$\text{Magnification} = \left(\frac{d_n}{f_e} + 1\right)\left(\frac{q_o}{f_o} - 1\right)$$

where q_o is the distance from the objective lens to the image it forms. Usually, q_o is close to 18 cm.

Telescope

A **telescope** that has an objective lens (or mirror) with focal length f_o and an eyepiece with focal length f_e gives a magnification $M = f_o / f_e$.

Interference and Diffraction of Light

Coherent Waves

Coherent waves are waves that have the same form, the same frequency, and a fixed phase difference (i.e., the amount by which the peaks of one wave lead or lag those of the other wave does not change with time).

The **relative phase** of two coherent waves traveling along the same

line together specifies their relative positions on the line. If the crests of one wave fall on the crests of the other, the waves are *in-phase*. If the crests of one fall on the troughs of the other, the waves are 180° (or one-half wavelength) *out-of-phase*.

Interference Effects

Interference effects occur when two or more coherent waves overlap. If two coherent waves of the same amplitude are superposed, *total destructive interference* (cancellation, darkness) occurs when the waves are 180° out-of-phase. *Total constructive interference* (reinforcement, brightness) occurs when they are in-phase.

Diffraction

Diffraction refers to the deviation of light from straight-line propagation. It usually corresponds to the bending or spreading of waves around the edges of apertures and obstacles. Diffraction places a limit on the size of details that can be observed optically.

Single-Slit Diffraction

When parallel rays of light of wavelength λ are incident normally upon a slit of width D, a diffraction pattern is observed beyond the slit. Complete darkness is observed at angles $\theta_{m'}$ to the straight-through beam, where

$$m'\lambda = D \sin \theta_{m'}$$

Here, $m' = 1, 2, 3, \ldots$, is the *order number* of the diffraction dark band.

Limit of Resolution

The **limit of resolution** of two objects due to diffraction:

If two objects are viewed through an optical instrument, the diffraction patterns caused by the aperture of the instrument limit our ability to distinguish the objects from each other. For distinguishability, the angle θ subtended at the aperture by the objects must be larger than a critical value θ_{cr}, given by

$$\sin \theta_{cr} = (1.22)\frac{\lambda}{D}$$

where D is the diameter of the circular aperture.

Diffraction Grating Equation

A **diffraction grating** is a repetitive array of apertures or obstacles that alters the amplitude or phase of a wave. It usually consists of a large number of equally spaced, parallel slits or ridges; the distance between slits is the grating spacing a. When waves of wavelength λ are incident normally upon a grating with spacing a, maxima are observed beyond the grating at angles θ_m to the normal, where

$$m\lambda = a \sin \theta_m$$

Here, m = 1, 2, 3, . . ., is the *order number* of the diffracted image.

This same relation applies to the major maxima in the interference patterns of even two and three slits. In these cases, however, the maxima are not nearly so sharply defined as for a grating consisting of hundreds of slits. The pattern may become quite complex if the slits are wide enough so that the single-slit diffraction pattern from each slit shows several minima.

Diffraction of X-Rays

The **diffraction of x-rays** of wavelength λ by reflection from a crystal is described by the *Bragg equation*. Strong reflections are observed at grazing angles ϕ_m (where ϕ is the angle between the face of the crystal and the reflected beam) given by

$m\lambda = 2d \sin \phi_m$

where d is the distance between reflecting planes in the crystal, and m = 1, 2, 3, . . ., is the *order* of reflection.

Optical Path Length

In the same time that it takes a beam of light to travel a distance d in a material of index of refraction n, the beam would travel a distance nd in air or vacuum. For this reason, nd is defined as the **optical path length** of the material.

Solved Problems

Solved Problem 6.1 What is the critical angle for light passing from glass (n = 1.54) to water (n = 1.33)?

Solution.

$n_1 \sin \theta_1 = n_2 \sin \theta_2$ becomes $n_1 \sin \theta_c = n_2 \sin 90^o$

from which

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1.33}{1.54} = 0.864 \text{ or } \theta_c = 59.7^o$$

Solved Problem 6.2 A camera gives a clear image of a distant landscape when the lens is 8 cm from the film. What adjustment is required to get a good photograph of a map placed 72 cm from the lens?

Solution. When the camera is focused for distant objects (for parallel rays), the distance between lens and film is the focal length of the lens, 8 cm. For an object 72 cm distant: