Relativity

<u>Outline :</u>

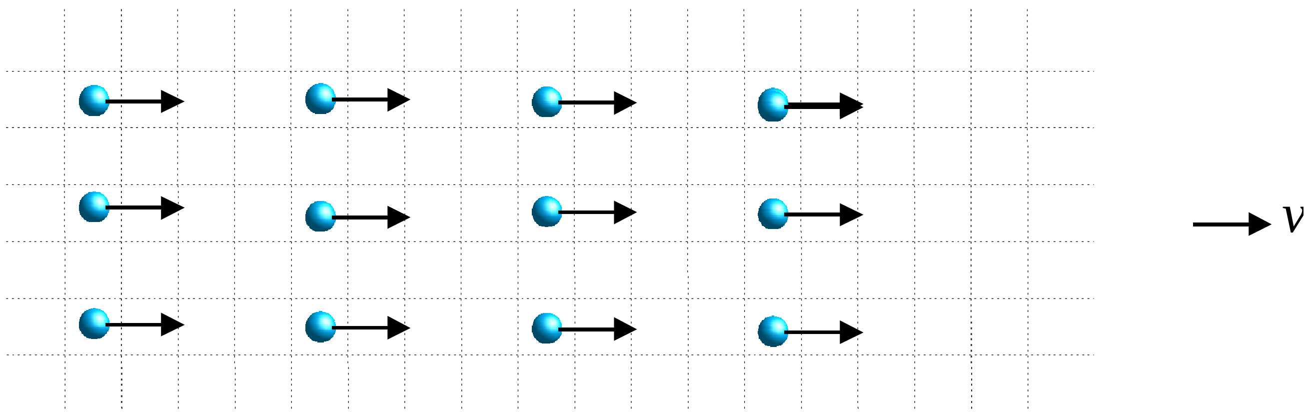
- The Principle of Relativity.
- Galilean Relativity.
- Electromagnetism & Relativity.
- Special Relativity : Lorentz Transformations.
- Special Relativity : Consequences.
- Relativistic Energy & Momentum.
- Anti−matter.

Principle of Relativity

- Symmetries are fundamental to physics.
- For example, the results of an experiment do not depend on the spatial orientation of the laboratory : **isotropy**.
- The principle of relativity refers to the symmetry between observers in co−moving inertial reference frames.

Inertial reference frame :

- A frame in which test particles with no external forces acting upon them move at constant speed in a straight line.



- The frame in which the imaginary array of particles is at rest is also an inertial frame, with a velocity relative to frame S of $\boldsymbol{v}$. All inertial frames are related to each other in this way.

Principle of Relativity

- With this definition, the principle of relativity takes the following simple form :

> The laws of physics are the same in all inertial reference frames

Weak :
Only the ***form*** of the physical law is invariant.

Strong :
Not only the ***form*** but the ***constants*** appearing in our physical laws are invariant.

↓

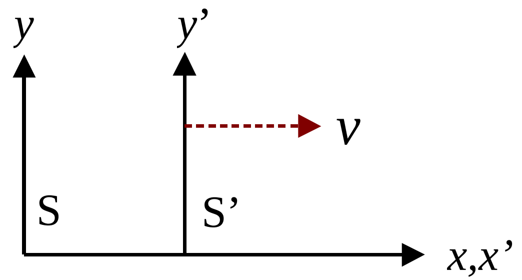
This is what we currently assume.

Galilean Relativity

Newton's Laws :

(1) Objects remain at rest or in a state of uniform motion unless acted upon by an external force.
(2) Force = mass × acceleration
(3) Action and reaction are equal and opposite.

Galilean Relativity :



- We know the force law in frame **S** :

$$F = ma = m \times \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$

- For an observer in a relatively moving frame **S'**:

$$x' = x - vt \;\Rightarrow\; \frac{\mathrm{d}x'}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t} - v \;\Rightarrow\; \frac{\mathrm{d}^2 x'}{\mathrm{d}t^2} = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$

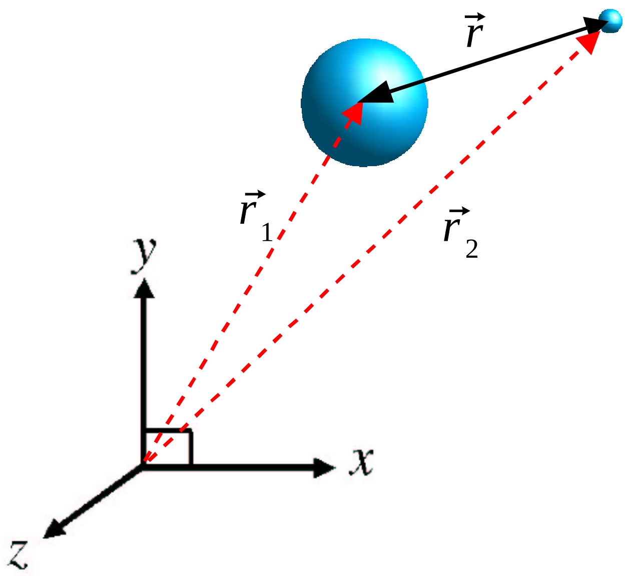
- Assuming the same force in both frames :

$$F' = F = m \times \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = m \times \frac{\mathrm{d}^2 x'}{\mathrm{d}t^2} = ma' \;\Rightarrow$$

Force law is the same in all inertial frames.

Galilean Relativity

- It also seemed natural that forces were invariant between different inertial frames :



$$F_{\text{GRAVITY}} = \frac{G M_1 M_2}{r^2}$$

(directed along the line between the two bodies)

But $\quad r^2 = \vec{r}\,.\vec{r} = (\vec{r}_1 - \vec{r}_2)\,.(\vec{r}_1 - \vec{r}_2)$

Under Galilean transformations :

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$

the separation is invariant :

$$r'^2 = r^2$$

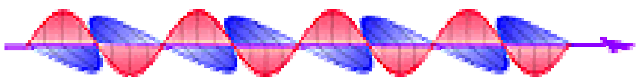
hence the force is the same in all inertial frames.
This is a consequence of the force **NOT** depending on velocity.

- In Newtonian mechanics, Galilean relativity is regarded as a *consequence* of the underlying theory. The modern view is that invariance under transformations between inertial reference frames is a *pre−requisite* of any theory.

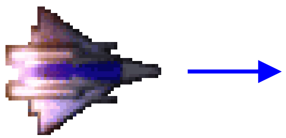
Electromagnetism

- As soon as Maxwell's equations of electromagnetism were formulated, it was clear that they were not invariant under Galilean transformations.

They predict a constant light speed in vacuum, ***c***. But according to the Galilean transformations, I ought to be able to "chase a light beam" and so end up in a frame in which the light travels at a speed different to ***c***.



Speed = ***c***



Speed = ***v***

⇒ In pilot's restframe, light would have Speed = ***c*** −***v***

The Lorentz force law :

$$\vec{F}_{\text{ELECTROMAGNETIC}} = q\,(\vec{E} + \vec{v} \wedge \vec{B})$$

depends on the velocity of the moving charge, $\vec{v}$.

⇒ The proof of Galilean invariance that we saw for Newton's gravity will necessarily fail.

Electromagnetism

- In the face of the non−invariance of Maxwell’s equations under Galilean transformations, we have two choices :

<u>Abandon Relativity</u>

- There is a preferred rest−frame in the universe, in which the speed of light is ***c*** and is isotropic. This is the rest frame of the "luminiferous ether" which carries electromagnetic waves.
- Maxwell’s equations are true only in this special frame.
- Search for the ether (see advanced topic).

<u>Impose Relativity</u>

- Find different transformations between inertial reference frames that preserve Maxwell’s equations and the speed of light.
- These transformations, the **Lorentz transformations**, were known at the same time as Maxwell’s equations, but their interpretation was unclear.
- Einstein in 1905 provided the first clear space−time interpretation.

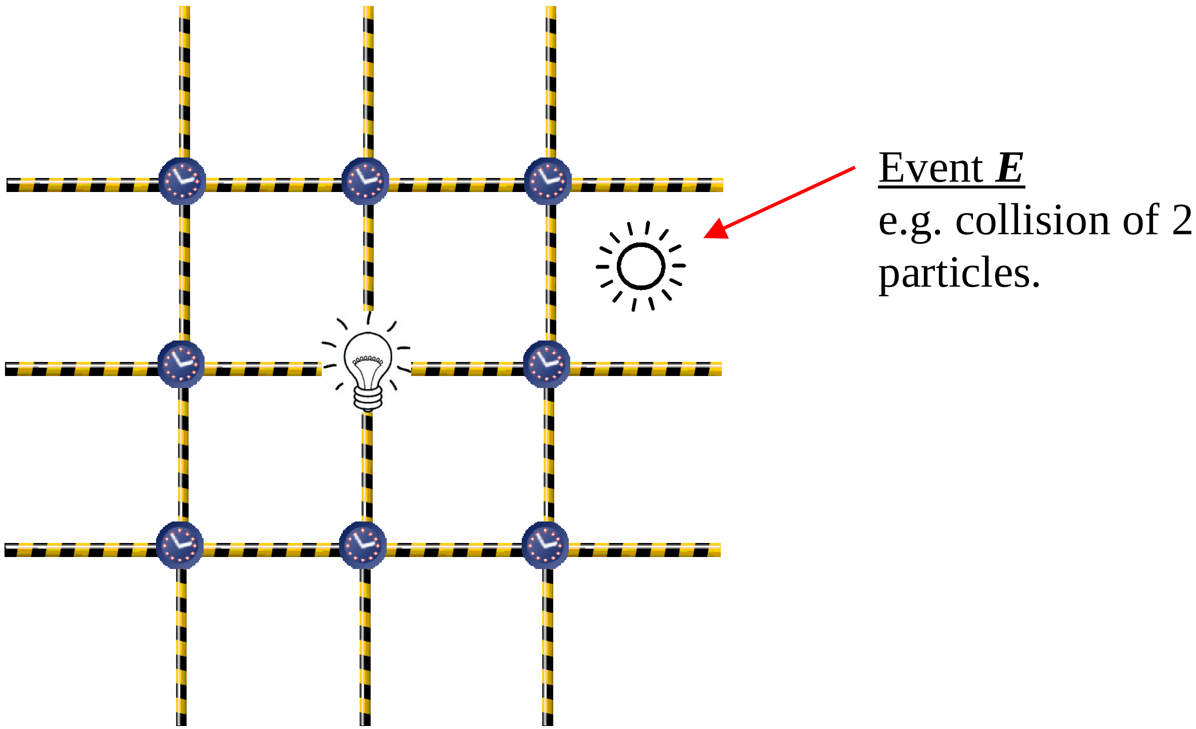
Light Signals & Coordinate Systems

- Since our Newtonian concepts of space & time will have to be modified, it's helpful to have a very clear procedure for mapping out space−time.

  1) Construct a 3−dimensional grid of rulers.

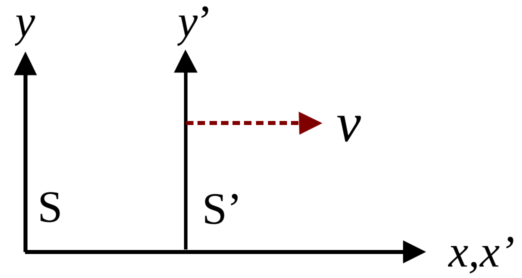
  2) Turn on a light at the agreed origin ($\boldsymbol{x,y,z,t}$) = (0,0,0,0). When the light pulse reaches a clock distance $\boldsymbol{d}$ from the origin, set the clock to read $\boldsymbol{t = d/c}$.

- This imaginary grid of clocks and rulers serves to uniquely identify the space−time co−ordinates of any event $\boldsymbol{E}$ in the given inertial frame.



Lorentz Transformations

- We need to go back to the drawing board and write down the most general coordinate transformations between co−moving frames :



- $x = vt$ describes the motion of the origin of S', i.e. (***x'*** = 0). Hence we can write quite generally :

  ① $x' = \gamma\,(x - vt)$ where γ is unknown

- The coordinate transformations from S to S' must be of the same form as those from S' to S. This means that if I interchange the primed and unprimed coordinates, and also substitute $\boldsymbol{v} \rightarrow -\boldsymbol{v}$, the same equations must hold :

  ② $x = \gamma\,(x' + vt')$

- A light pulse along the joint ***x***, ***x'***−axis must travel with speed ***c*** in both frames :

  $x = ct$ and $x' = ct'$

<u>Einstein's law of light propagation</u>

Lorentz Transformations

- Substituting the light pulse coordinates into the transformation equations ① and ② :

$$x' = ct' = \gamma(x - vt) = \gamma(ct - vt)$$
$$x = ct = \gamma(x' + vt') = \gamma(ct' + vt')$$

- Multiplying and dividing through by tt' gives :

$$c^2 = \gamma^2 (c^2 - v^2)$$

$$\Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Lorentz γ Factor

⇒ Approaches 1 in the non−relativistic limit $v \ll c$.

- Eliminating x' from ① and ② gives the remaining transformation equation :

$$t' = \gamma \left(t - vx/c^2\right)$$

Lorentz Transformations

- Summarising :

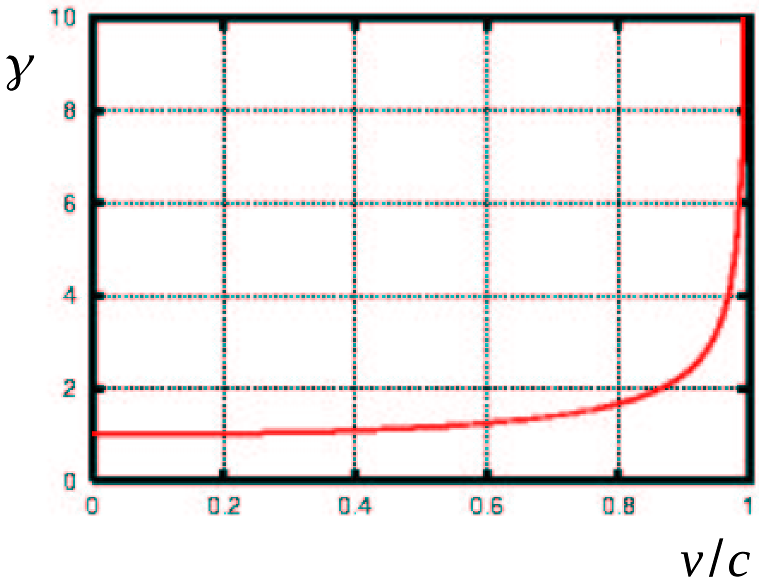
$$x' = \gamma\,(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\,(t - vx/c^2)$$

Lorentz Transformations S→S'

$$x = \gamma\,(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\,(t' + vx'/c^2)$$

Lorentz Transformations S'→S

⇒ Approach Galilean transformations in the non−relativistic limit $v \ll c$.



Lorentz Transformations

- The most important feature of the Lorentz transformations is that the following quantity :

$$c^2 t^2 - x^2 - y^2 - z^2$$

is the same in all frames. It is a **Lorentz invariant** quantity.

- These play a particularly important role, since they are the only quantities agreed upon by all observers.
- Once again, coordinate intervals satisfy the same equations :

$$c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

is also an invariant.

<u>Proof :</u> substitute the Lorentz transformation equations into the expression :

$$c^2 t'^2 - x'^2 - y'^2 - z'^2$$

and note that :

$$\gamma^2 \times \left(1 - \frac{v^2}{c^2}\right) = 1$$

The Speed of Light as a Limiting Speed

- So far we have used the *constancy* of the speed of light to derive the Lorentz transformations.
- Relativity also implies that the speed of light is a *limiting* speed in physics.

☆ <u>Kinematics.</u> Super−luminal velocities would allow travel backwards in time.

$$\Delta t' = \gamma \left(\Delta t - \frac{v \Delta x}{c^2}\right)$$

$$\frac{\Delta t'}{\Delta t} = \gamma \left(1 - \frac{v}{c} \frac{\Delta x / \Delta t}{c}\right)$$

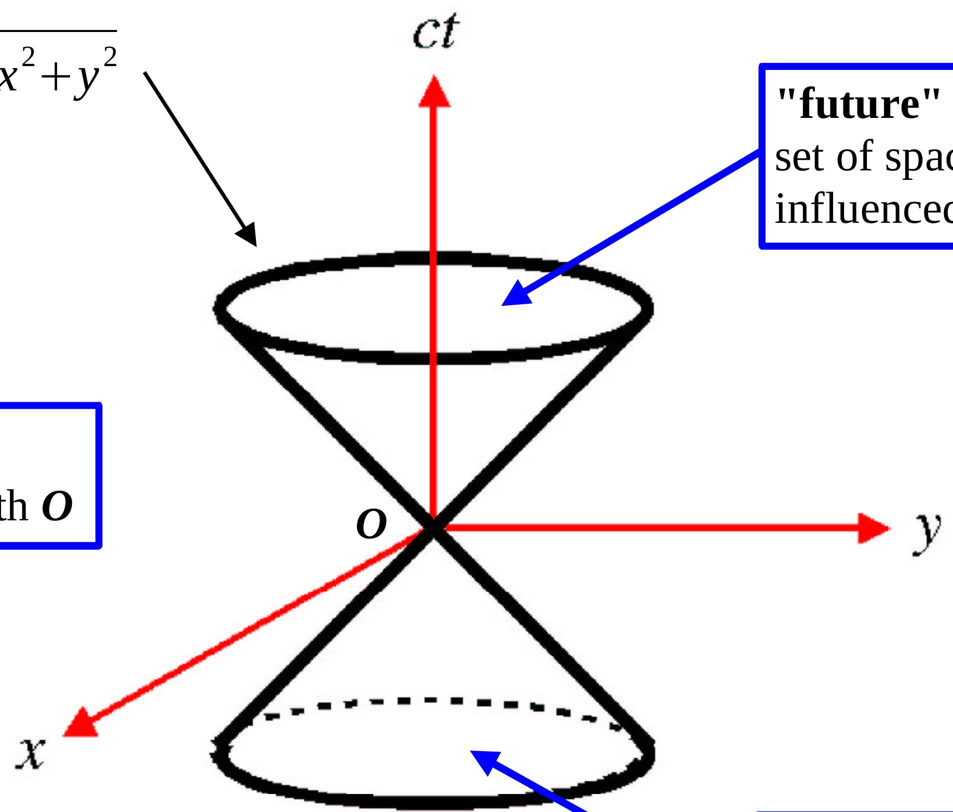
speed of an object in frame S

- If $\Delta x / \Delta t > c$, this term can be negative.
- Then $\Delta t'$, the corresponding time interval in S', can be negative.
- Time travel paradoxes.
- See graphical derivation of this result in advanced topics.

☆ <u>Dynamics.</u> It would take infinite energy to accelerate a massive object to the speed of light.

Light Cones

$ct = \sqrt{x^2+y^2}$



"future" :
set of space−time points influenced by ***O***

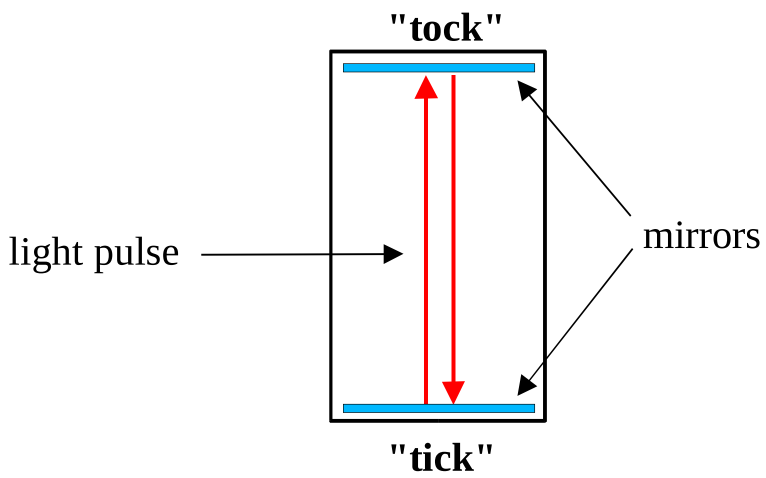
"present" :
no causal contact with ***O***

No unique definition of simultaneity. The end of the concept of absolute time (see advanced notes).

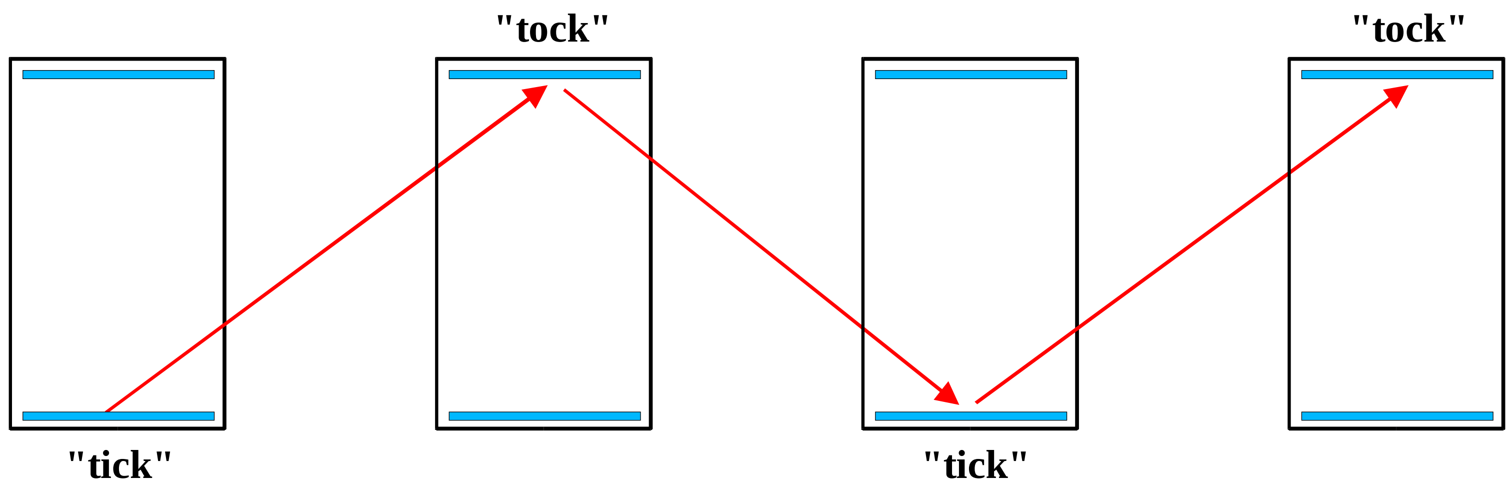
"past" :
set of space−time points that can influence ***O***

Time Dilation

- Consider a "light clock" :



stationary clock



moving clock : light travels further between "ticks" *at the same speed*.

Time Dilation

- **Moving clocks run slow**.
- Not just a peculiarity of light clocks : any physical time−keeping process must give the same result.
- The can be seen directly from the Lorentz transformations :
- Start from : $t = \gamma\,(t' + vx'/c^2)$

$$\Delta t = \gamma\,(\Delta t' + v\,\Delta x'/c^2)$$

- The moving clock is stationary in frame S' $\Rightarrow$ $\Delta x' = 0$

$$\boxed{\Delta t = \gamma\,\Delta t'}$$

➨ Reminder : $\gamma = 1/\sqrt{1-v^2/c^2}$

Proper Time

- The time elapsed in the rest frame of an object is called the **proper time**.
- The proper time is the amount of time that the object has "aged", regardless of the time indicated in other inertial frames.

person's rest frame	**1 Jan 2000**	**…**	**2 Jul 2090**
some other inertial frame	**1 Jan 2000**	**…**	**4 Oct 3052**

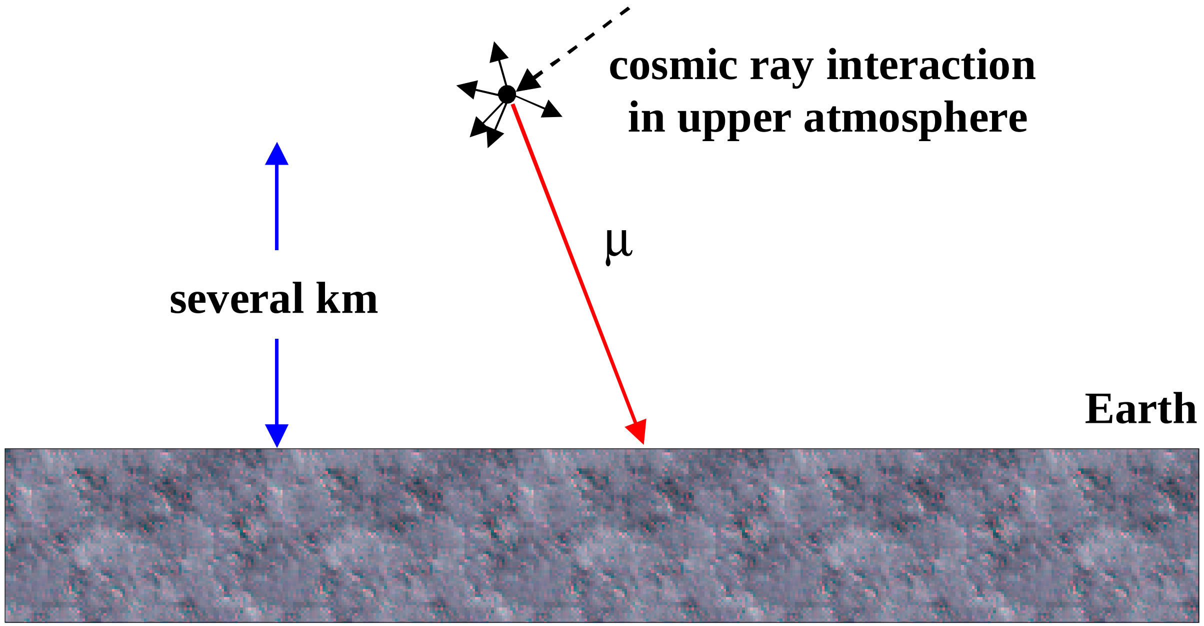
- Note : the elapsed proper time multiplied by c^2 is just the invariant quantity $c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$ evaluated in the object's rest frame.

Proper Time

- Another example, cosmic ray muons have a mean **proper** lifetime (τ) of 2 μs.
  - Muons at rest decay after an average of 2 μs.
  - Without time dilation, they would travel roughly 2 $\mu s \times c$ = 600 metres.
  - In fact, highly relativistic muons (i.e. speeds close to the speed of light) travel a distance :

$$d \approx \gamma \times \tau \times c$$

  - In this case, $\gamma >> 1$ and the muons can travel many kilometres before decaying.



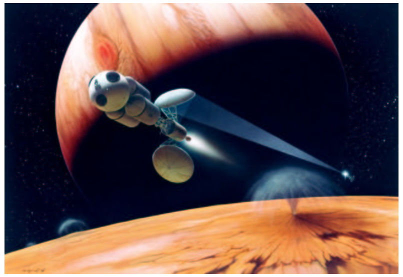
Time Travel

- Time dilation implies that it is possible to travel into the future.
- You just have to travel fast !

Synchronise two clocks before leaving clock S on earth and taking clock S’ on a space voyage.

S: 00:00.00

S’: 00:00.00



S: 02:00.00

S’: 01:59.00

S: 04:00.00

S’: 03:58.00

When the travelling clock arrives back, it reads an earlier time than the clock left on earth.

- However, the effect is usually tiny, e.g. Apollo spacecraft to the moon travelled at ≈ 10,000 ms^{-1} :

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1.0000000005$$

➨ Clocks on earth would have gained 1 second after a journey of 60 years !

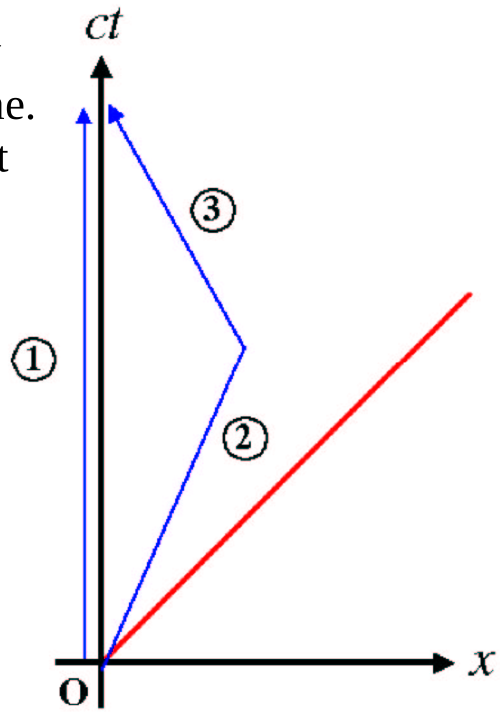
Twin Paradox

- Doesn't this lead to a paradox ?

  - An earthbound twin sees his astronaut twin's clock run slow, and therefore expects his astronaut twin to arrive back younger.
  - The astronaut's rest frame is (by the relativity principle) just as valid an inertial frame, so the space faring twin sees his earthbound twin's clock run slow.
  - They both expect their sibling to be younger when they next meet.
  - They cannot both be right !

- But : the earthbound twin's journey takes places in a single inertial frame. The astronaut twin's journey is split into two separate inertial phases

  .



Harder :

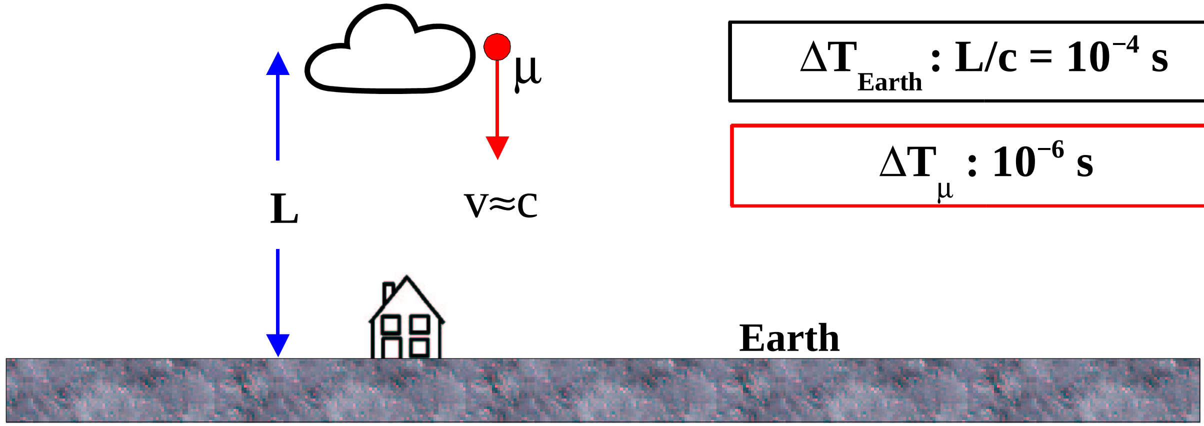
You can calculate the proper times elapsed during the two journeys using the invariant :

$$c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

evaluated on each leg separately.

Length Contraction

- Earth's rest frame :



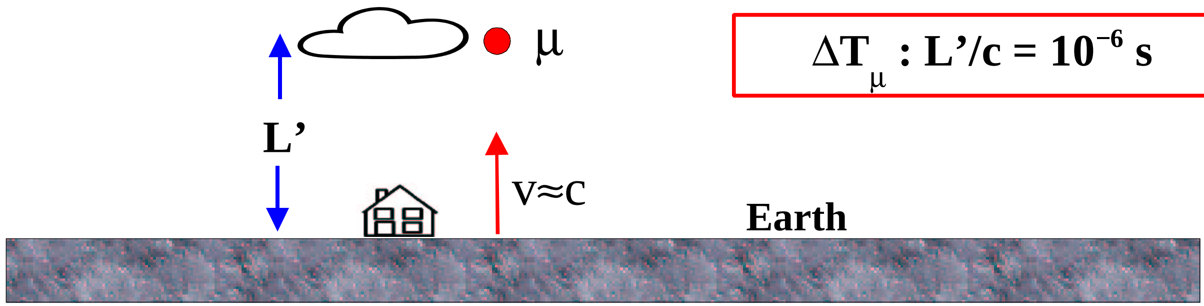
ΔT_{Earth} : $L/c = 10^{-4}$ s

ΔT_{μ} : 10^{-6} s

$\Delta T_{\mu} = \Delta T_{Earth}/\gamma$

Time Dilation

- Muon's rest frame :



ΔT_{Earth} : $L/c = 10^{-4}$ s

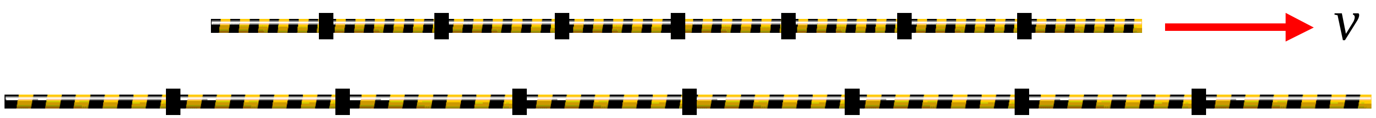
ΔT_{μ} : $L'/c = 10^{-6}$ s

$\Rightarrow L' = L/\gamma$

Length Contraction

Length Contraction

- Moving rulers look short :



- Starting from the Lorentz transformations :

$$x' = \gamma\,(x - vt)$$

$$\Rightarrow \quad \Delta x' = \gamma\,(\Delta x - v\,\Delta t)$$

- Then according to definition (1), $\Delta t = 0$:

$$\boxed{\Delta x = \frac{\Delta x'}{\gamma}}$$

Relativistic Energy & Momentum

- Clearly our existing expressions for momentum and kinetic energy will require modification :

$$p = mv$$

$$E = \frac{1}{2}mv^2 + C$$

experiments are not sensitive to such a constant, but it is allowed.

- Since $v < c$, energies and momenta would be limited to certain values. This is inconsistent, since work can still be done on relativistic particles.

- Einstein suggested the following modifications to our definitions :

$$p = \gamma m_0 v$$

$$E^2 = p^2c^2 + m_0^2c^4$$

m_0 is a constant, the body's mass while at rest

"rest energy"

- Since γ grows as $v \to c$, the momentum and energy can continue to increase.

$\mathbf{E = mc^2}$

- How does this relate to the famous equation $E = mc^2$? The equation :

$$E^2 = p^2 c^2 + m_0^2 c^4$$

is exactly the same as writing :

$$E = m\, c^2$$

so long as :

$$\boxed{m = \gamma\, m_0}$$

inertial mass   *rest* mass

$E = mc^2$

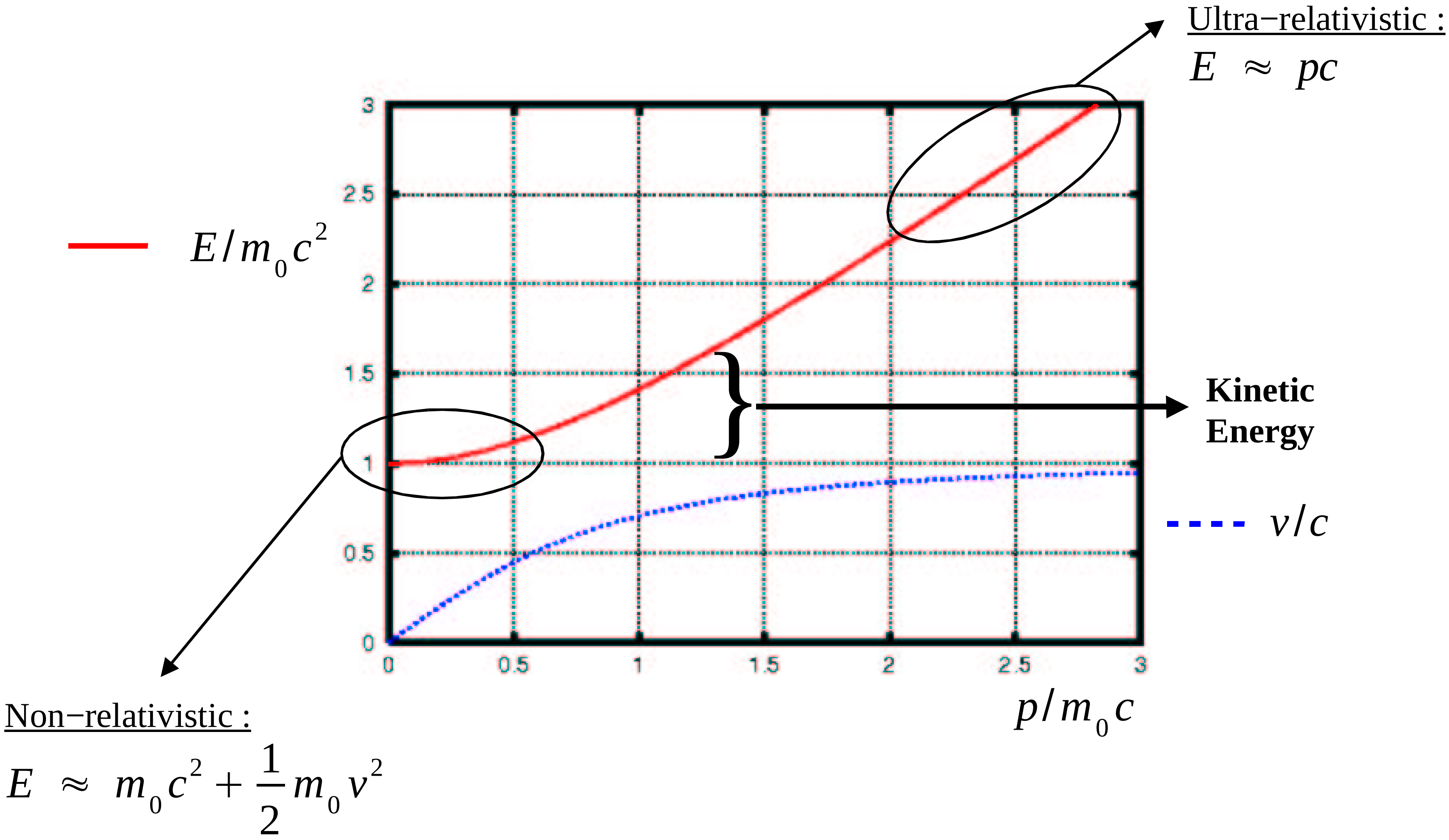
- Comparison between Newtonian and relativistic kinematics :

Animation

$\Rightarrow$ **constant force**

Particles start at rest with *m* (inertial mass) equal to 1.0

$E = mc^2$



Ultra−relativistic :
$E \approx pc$
E/m_0c^2
Kinetic Energy
v/c
p/m_0c
0
0.5
1
1.5
2
2.5
3
Non−relativistic :
$E \approx m_0c^2 + \frac{1}{2}m_0v^2$

Antiparticles

- The fundamental equation of relativistic dynamics :

$$E^2 = p^2c^2 + m_0^2c^4$$

permits negative as well as positive energy solutions :

$$E = \pm\sqrt{p^2c^2 + m_0^2c^4}$$

- We often ignore certain solutions as being "unphysical" in solving physics problems.
- BUT, it turns out that quantum mechanics forces us to consider the negative energy solutions. It requires BOTH wavefunctions :

$$\psi \sim \psi(\text{space}) \times e^{iEt/\hbar}$$
$$\psi \sim \psi(\text{space}) \times e^{-iEt/\hbar}$$

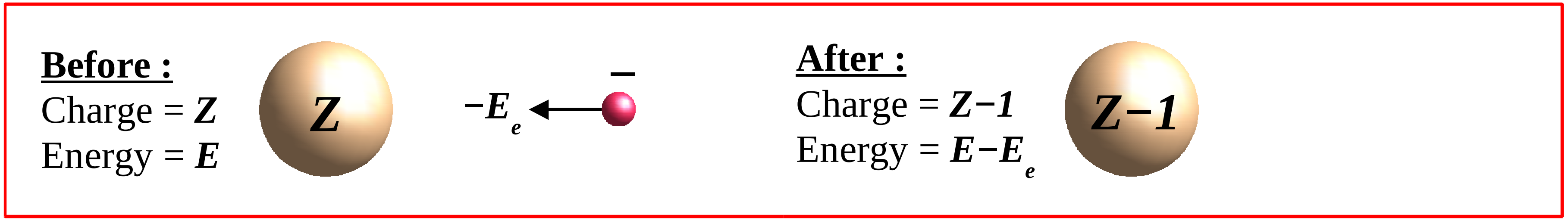
to exist.

➨ We are forced to provide a physical interpretation for the negative energy solutions.

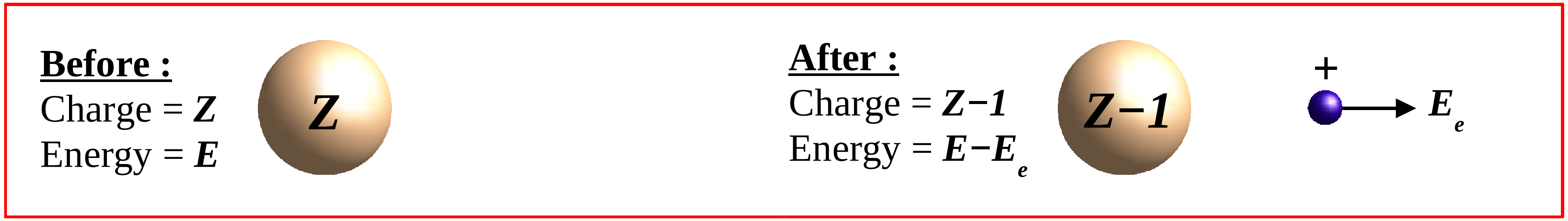
Antiparticles

- But a (negatively charged) electron with **negative** energy and momentum behaves exactly like a (positively charged) positron with **positive** energy and momentum.

- For example, imagine a nucleus absorbing a negative energy electron :



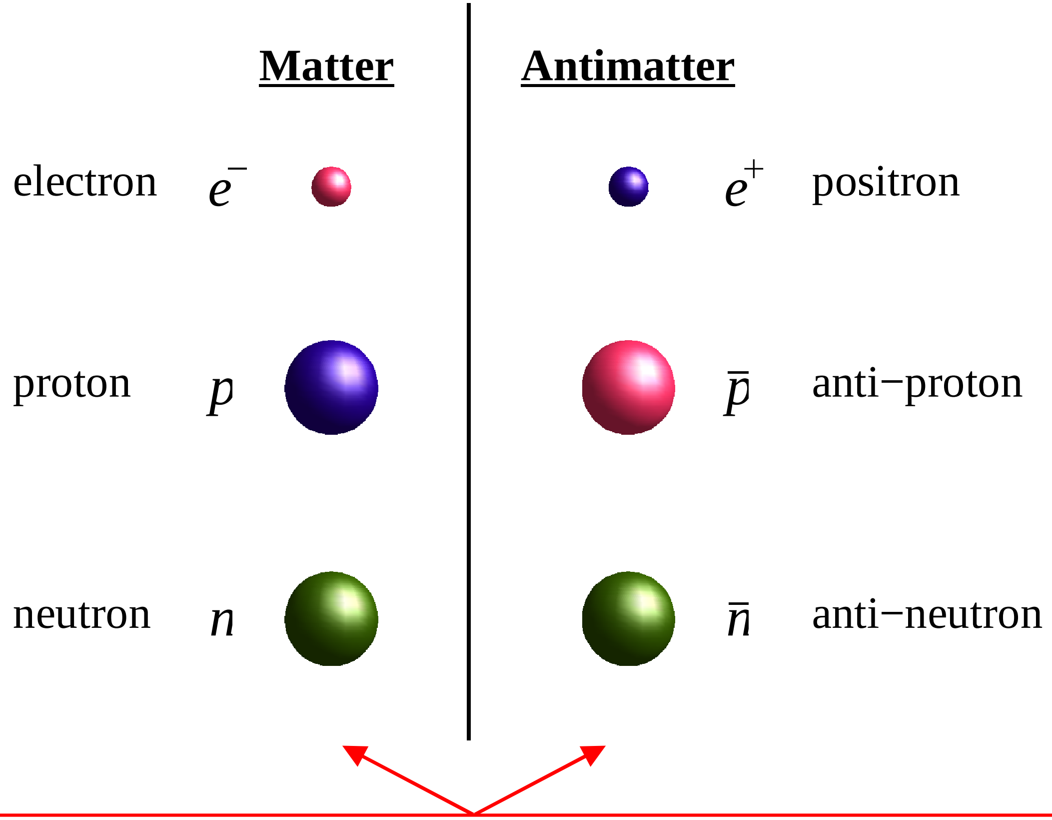
- This is exactly the same as emitting a positive energy positron :



- Some nuclei (e.g. ^{64}Cu) decay in exactly this manner.

Antiparticles

- So, relativity and quantum mechanics together predict the existence of antimatter particles corresponding to every known matter particle.



- Same mass and size
- Opposite charge (and some other "quantum numbers")

Summary

- The principle of relativity states that :

> The laws of physics are the same in all inertial reference frames

- Newton's laws are invariant under Galilean transformations between inertial reference frames :

$$
\begin{aligned}
x' &= x - vt \\
y' &= y \\
z' &= z \\
t' &= t
\end{aligned}
$$

- The laws of electromagnetism, with their prediction of a constant light speed, are not invariant under Galilean transformations, but are invariant under the Lorentz transformations :

$$
\begin{aligned}
x' &= \gamma\,(x - vt) \\
y' &= y \\
z' &= z \\
t' &= \gamma\,(t - vx/c^2)
\end{aligned}
\qquad \text{where : } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}
$$

Summary

- Many highly counter−intuitive consequences flow from these transformations :
  - Different observers do not agree on which spatially separated events are simultaneous. The end of Newtonian absolute time.
  - Moving clocks run slow by a factor of γ .
  - Moving objects appear contracted by the same factor γ .
- The relativistic formula for energy and momentum :

$$E = mc^2 = \gamma m_0 c^2$$

tells us that mass and energy are equivalent. Mass can be converted into energy (nuclear reactor) and energy can be converted into mass (particle accelerator).

- The full formula relating relativistic energy and momentum :

$$E^2 = p^2 c^2 + m_0^2 c^4$$

permits negative energy solutions and gives us the first indication of the presence of "anti−matter".

Exercises

1) A train 110m long passes a station platform 100m in length. Use the length contraction formula $L' = L/\gamma$ to calculate how fast the train must be moving in order to fit into the station. What happens if the train stops to let passengers off ?

2) A time traveller is prepared to leave earth and travel for 1 year of his own time, in order to arrive back on earth 2 years into the future :

   i. How fast must he travel ?

   ii. What energy is required to accelerate his spaceship to the required speed (as a multiple of its rest mass energy) ?

Exercises

3) The nuclear reaction powering the sun is :

$$4\left({}^{1}\mathrm{H}\right) \rightarrow {}^{4}\mathrm{He} + 2e^{+} + 2 \ \text{neutrinos} + \text{energy}$$

The masses of the hydrogen and helium atoms are :

$$m(\mathrm{H}) = 1.00794 \ \ \text{amu}$$

$$m(\mathrm{He}) = 4.002602 \ \ \text{amu}$$

where $1 \ \text{amu} = 1.66\times10^{-27} \ \text{kg}$

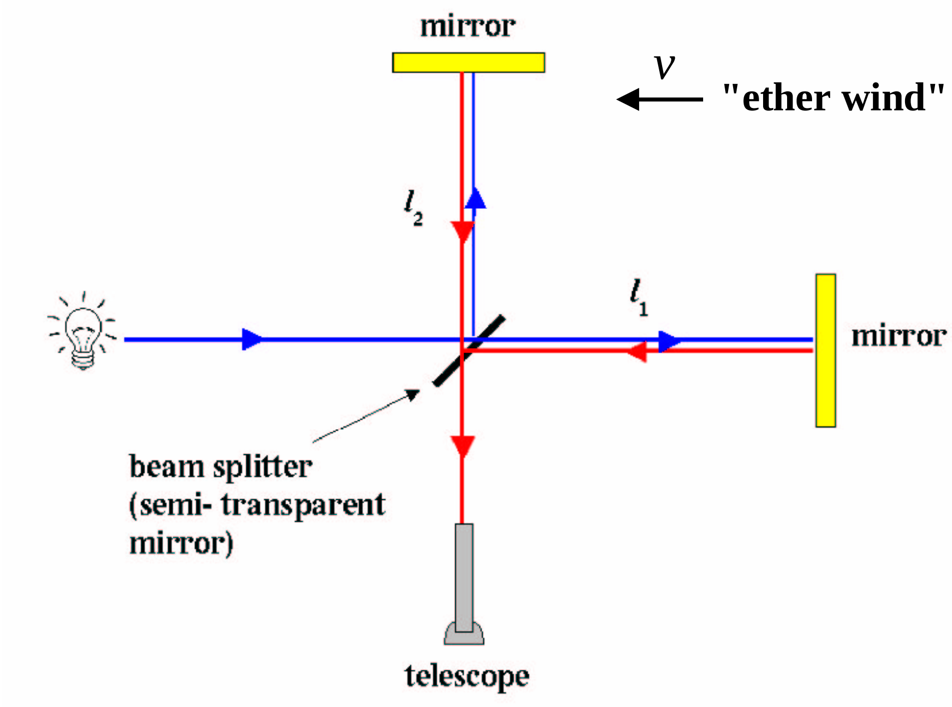
i. What is the energy liberated in this reaction ? (You can ignore the positrons and neutrinos)

ii. Given that this reaction takes place 10^{38} times per second, at what rate is energy generated inside the sun ?

Ether Searches (*advanced*)

Michelson−Morley experiment :

The difference in path length between light travelling along the perpendicular arms gives rise to an interference pattern at the telescope.



Calculate the time taken for light to travel down each arm of the spectrometer and back in the presence of an ether wind :

$$\text{Arm 1}: \; t_1 = \frac{l_1}{c-v} + \frac{l_1}{c+v}$$

outward **return**

$$\text{Arm 2}: \; t_2 = \frac{l_2}{\sqrt{c^2-v^2}} + \frac{l_2}{\sqrt{c^2-v^2}}$$

After some algebra : $\Delta \approx t_1 - t_2 \approx \frac{2(l_1-l_2)}{c} + \frac{2l_1 v^2}{c^3} - \frac{l_2 v^2}{c^3}$

Note : to go through the algebra, you will need to remember the Binomial Theorem :

$(1-v^2/c^2)^n = 1 - n\,v^2/c^2 - \ldots$ and ignore all higher order terms. This is OK if $v \ll c$

Ether Searches (*advanced*)

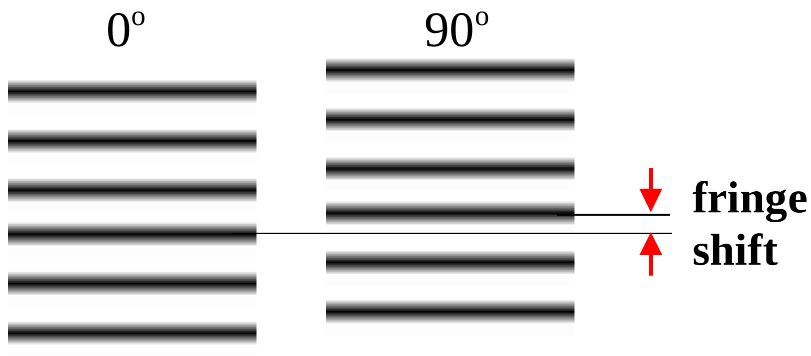
- Since the absolute lengths l_1 and l_2 are not well known, a direct measurement of v is difficult.
- If the apparatus is **rotated** by 90°, a similar expression for the time difference for light along the two arms can be derived :

$$\Delta' \approx t_1' - t_2' \approx \frac{2(l_1 - l_2)}{c} + \frac{l_1 v^2}{c^3} - \frac{2 l_2 v^2}{c^3}$$

- Then the difference between Δ and Δ' corresponds to a fringe shift between the two orientations of the interferometer. Assuming for simplicity that $l_1 = l_2$:

velocity of earth through ether

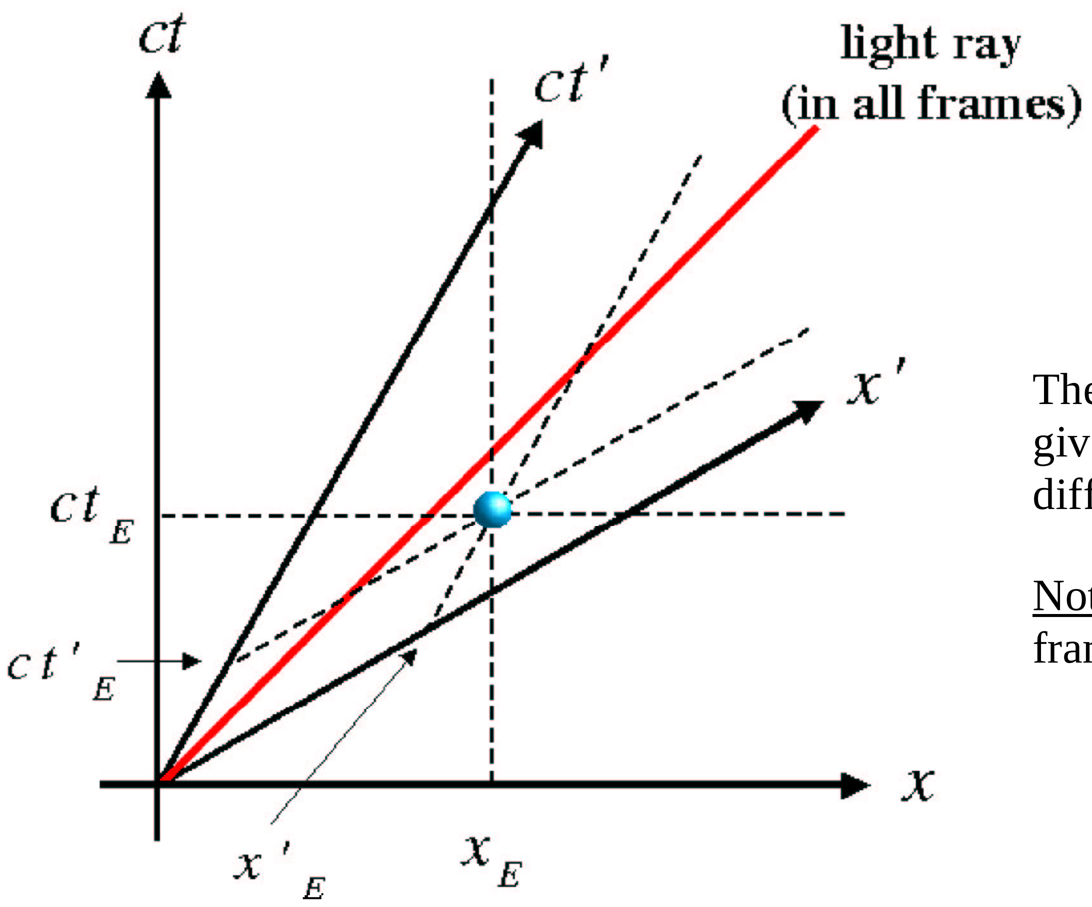
$$\text{fringe shift} = \frac{c\,(\Delta - \Delta')}{\lambda} = \frac{2\,(v/c)^2}{\lambda/l}$$



- Outcome : one of the most important null results in physics.
- Despite attempts to explain the null result in terms of "ether−drag" effects, there is no evidence for absolute motion with respect to an "ether". <u>There is no preferred inertial reference frame</u>.

Graphical Representation (*advanced*)

- The Lorentz transformations can be depicted graphically :

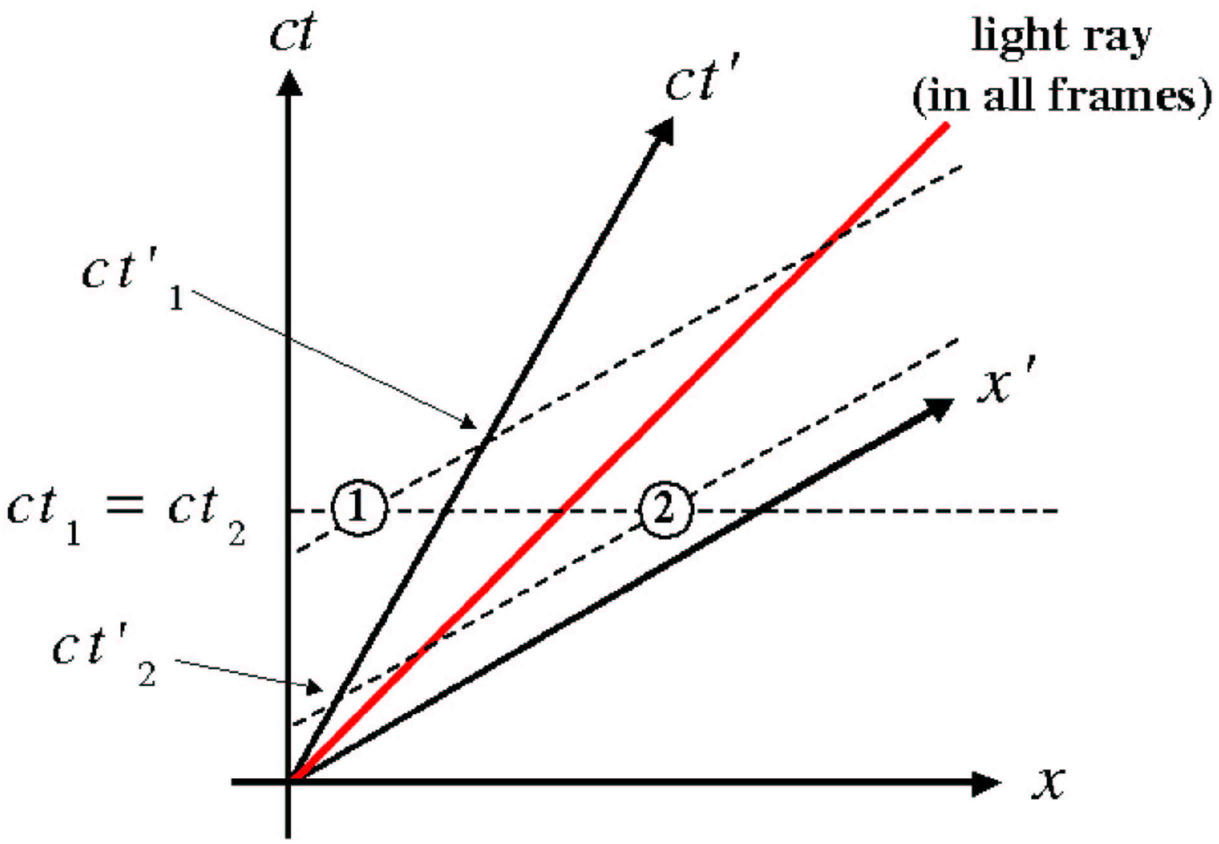


The space−time coordinates for a given event can be read off on different sets of axes.

<u>Note</u> : the **scales** on the axes for the frames S and S' are different !

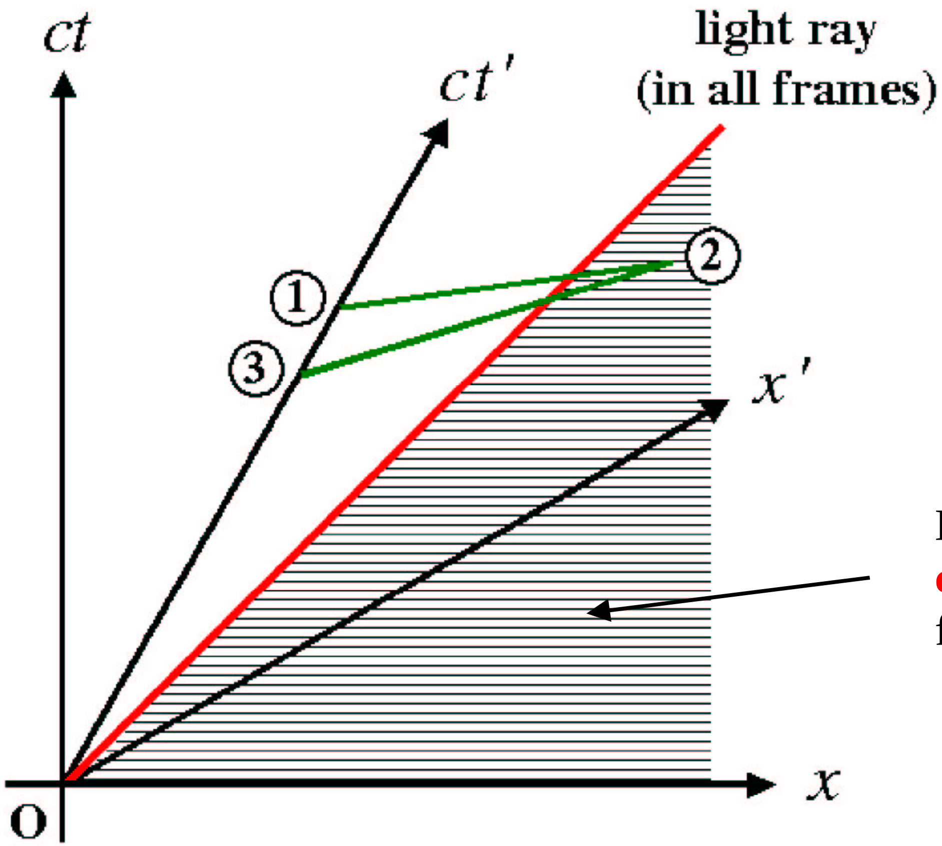
Relativity of Simultaneity (*advanced*)

- Spatially separated events simultaneous in S are in general not simultaneous in S'.
- The end of the Newtonian concept of absolute time.



Causal Contact (*advanced*)

- Super−luminal velocities in frame S correspond to travel backwards in time in another frame S'.
- This allows paradox generating journeys to become possible : ① → ② → ③ . I arrive back at the same place before I departed !
- ➡ The limiting velocity in the universe is **c** .



Events in this region are **causally disconnected** from the origin, **O**

Relativistic Energy & Momentum (*advanced*)

- We have mainly been dealing with *kinematics* : statements about fundamental measurements of space & time.
- Clearly our *dynamical* concepts of energy & momentum will need modification.
- Unlike Newtonian mechanics, the concept of a force is a difficult place to start.
- We should start from a quantity that we know is invariant (does not depend on the frame of reference) :

$$c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

- Another quantity that we know is invariant is the proper time interval $\Delta\tau$. Therefore :

$$c^2 \frac{\Delta t^2}{\Delta \tau^2} - \frac{\Delta x^2}{\Delta \tau^2} - \frac{\Delta y^2}{\Delta \tau^2} - \frac{\Delta z^2}{\Delta \tau^2}$$

- But we know from the formula for time dilation that :

$$\Delta t / \Delta \tau = \gamma$$

and it's also harmless to multiply by a constant $m_0^2 c^2$, where m_0 is the object's mass :

$$\gamma^2 m_0^2 c^4 - m_0^2 c^2 \left(\frac{\Delta x^2}{\Delta \tau^2} + \frac{\Delta y^2}{\Delta \tau^2} + \frac{\Delta z^2}{\Delta \tau^2} \right)$$

Relativistic Energy & Momentum (*advanced*)

- So far we have :

$$\underbrace{\gamma^2 m_0^2 c^4}_{\textbf{energy}^2} - \underbrace{m_0^2 c^2 \left(\frac{\Delta x^2}{\Delta \tau^2} + \frac{\Delta y^2}{\Delta \tau^2} + \frac{\Delta z^2}{\Delta \tau^2}\right)}_{\textbf{momentum}^2}$$

- The final result is :

$$\boxed{E^2 - p^2 c^2 = m_0^2 c^4}$$

Get the constant just by evaluating the expression in the rest frame of the object.

$\Rightarrow$ $m_0^2 c^2$ is the rest energy.

- Relativistic energy & momentum are conserved just as Newtonian energy and momentum are. The above formulation guarantees that conservation in one frame implies conservation in all frames.

General Relativity (*advanced*)

- The equivalence of *inertial* and *gravitational* mass was first noted by Galileo :

$$M_I \times a = M_G \times g$$

inertial mass: M_I

gravitational mass: M_G

$$g = \frac{GM_{EARTH}}{r^2}$$

- Different objects only fall under gravity with the same acceleration if $\boldsymbol{M_I = M_G}$. This equality is accidental in Newtonian physics.
- Einstein elevated this apparent coincidence to an equivalence principle, which says, in effect that no experiment can tell the difference between a gravitational and an inertial field :

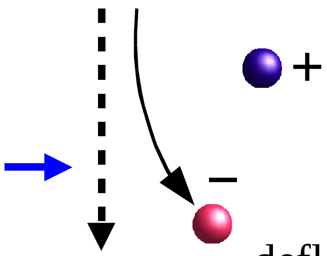
General Relativity (*advanced*)

- Alternative statement of the same principle : all freely−falling laboratories are equivalent for the performance of all physical experiments.
- Einstein's general theory guarantees this because it views all free−fall as just following the lines of (possibly curved) space−time. Mass is not viewed as the gravitational equivalent of electric charge, so there is no need to wonder why gravitational mass is the same as inertial mass.

<u>Electrostatics :</u>

<u>Source :</u> an electric charge creates a surrounding electric field.

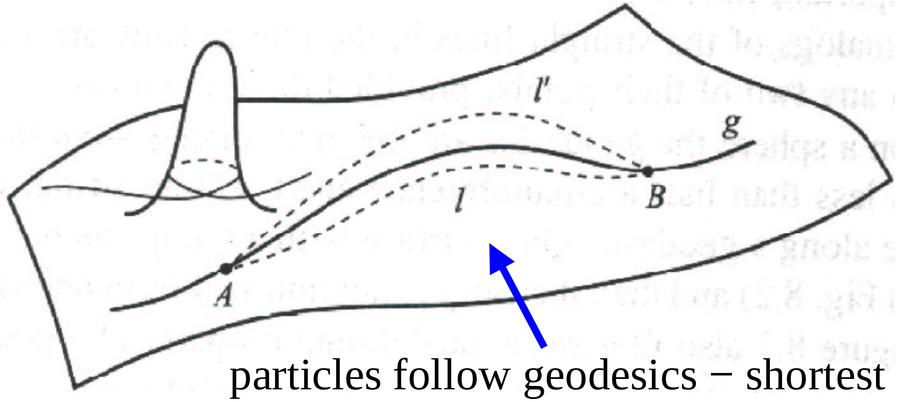
<u>Effect :</u> other electric charges feel the electric field as a force. The force causes them to deviate from their free−fall trajectories.



<u>Gravity :</u>

<u>Source :</u> masses create distortions in the surrounding space−time.

<u>Effect :</u> the shape of space−time determines the shape of the free−fall trajectories, which are followed by all freely falling bodies.



particles follow geodesics − shortest path between nearby points

General Relativity (*advanced*)